Miscellaneous Examples

Example 16 Solve $-8 \leq 5x - 3 < 7$.

Solution In this case, we have two inequalities, $-8 \leq 5x - 3$ and $5x - 3 < 7$, which we will solve simultaneously. We have $-8 \leq 5x - 3 < 7$

or $-5 \leq 5x < 10$ or $-1 \leq x < 2$

Example 17 Solve $-5 \leq \dfrac{5 - 3x}{2} \leq 8$.

Solution We have $-5 \leq \dfrac{5 - 3x}{2} \leq 8$

or $-10 \leq 5 - 3x \leq 16$ or $-15 \leq -3x \leq 11$

or $5 \geq x \geq -\dfrac{11}{3}$

which can be written as $\dfrac{-11}{3} \leq x \leq 5$

Example 18 Solve the system of inequalities:

$3x - 7 < 5 + x$ ... (1)

$11 - 5x \leq 1$ ... (2)

and represent the solutions on the number line.

Solution From inequality (1), we have

$3x - 7 < 5 + x$

or $x < 6$ ... (3)

Also, from inequality (2), we have

$11 - 5x \leq 1$

or $-5x \leq -10$ i.e., $x \geq 2$ ... (4)

If we draw the graph of inequalities (3) and (4) on the number line, we see that the values of x, which are common to both, are shown by bold line in Fig 6.15.
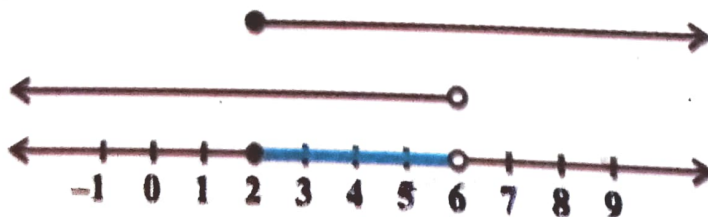

Fig 6.15

Thus, solution of the system are real numbers x lying between 2 and 6 including 2, i.e.,

$2 \leq x < 6$

Example 19 In an experiment, a solution of hydrochloric acid is to be kept between 30° and 35° Celsius. What is the range of temperature in degree Fahrenheit if conversion formula is given by $C = \dfrac{5}{9}$ (F – 32), where C and F represent temperature in degree Celsius and degree Fahrenheit, respectively.

Solution It is given that $30 < C < 35$.

Putting $\quad C = \dfrac{5}{9}$ (F – 32), we get

$$30 < \dfrac{5}{9} \text{ (F – 32)} < 35,$$

or $\quad \dfrac{9}{5} \times (30) < \text{(F – 32)} < \dfrac{9}{5} \times (35)$

or $\quad 54 < \text{(F – 32)} < 63$

or $\quad 86 < F < 95.$

Thus, the required range of temperature is between 86° F and 95° F.

Example 20 A manufacturer has 600 litres of a 12% solution of acid. How many litres of a 30% acid solution must be added to it so that acid content in the resulting mixture will be more than 15% but less than 18%?

Solution Let x litres of 30% acid solution is required to be added. Then

$$\text{Total mixture} = (x + 600) \text{ litres}$$

Therefore $\quad 30\% \, x + 12\% \text{ of } 600 > 15\% \text{ of } (x + 600)$

and $\quad 30\% \, x + 12\% \text{ of } 600 < 18\% \text{ of } (x + 600)$

or $\quad \dfrac{30x}{100} + \dfrac{12}{100} (600) > \dfrac{15}{100} (x + 600)$

and $\quad \dfrac{30x}{100} + \dfrac{12}{100} (600) < \dfrac{18}{100} (x + 600)$

or $\quad 30x + 7200 > 15x + 9000$

and $\quad 30x + 7200 < 18x + 10800$

or $\quad 15x > 1800 \text{ and } 12x < 3600$

or $\quad x > 120 \text{ and } x < 300,$

i.e. $\quad 120 < x < 300$

Thus, the number of litres of the 30% solution of acid will have to be more than 120 litres but less than 300 litres.

Summary

- Two real numbers or two algebraic expressions related by the symbols $<, >, \leq$ or $\geq$ form an inequality.

- Equal numbers may be added to (or subtracted from) both sides of an inequality.

- Both sides of an inequality can be multiplied (or divided) by the same positive number. But when both sides are multiplied (or divided) by a negative number, then the inequality is reversed.

- The values of x, which make an inequality a true statement, are called *solutions of the inequality*.

- To represent $x < a$ (or $x > a$) on a number line, put a circle on the number a and dark line to the left (or right) of the number a.

- To represent $x \leq a$ (or $x \geq a$) on a number line, put a dark circle on the number a and dark the line to the left (or right) of the number x.

- If an inequality is having $\leq$ or $\geq$ symbol, then the points on the line are also included in the solutions of the inequality and the graph of the inequality lies left (below) or right (above) of the graph of the equality represented by dark line that satisfies an arbitrary point in that part.

- If an inequality is having $<$ or $>$ symbol, then the points on the line are not included in the solutions of the inequality and the graph of the inequality lies to the left (below) or right (above) of the graph of the corresponding equality represented by dotted line that satisfies an arbitrary point in that part.

- The solution region of a system of inequalities is the region which satisfies all the given inequalities in the system simultaneously.